Pharmacy College

Mathematic & Biostatistics

Chapter Three

1st Stage…….1st Semester

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Chapter Three

Elementary probability concept:

Probability is a measure or estimation of likelihood of occurrence of an event. Probabilities are given a value between 0 (0% chance or *will not happen*) and 1 (100% chance or *will happen*). The higher the degree of probability, the more likely the event is to happen, or, in a longer series of samples, the greater the number of times such event is expected to happen.

Provide the means for measuring the uncertainty of decisions and inferences based on data from samples & experiments.
We will use common biomedical situation for instruction in the assessment and use of measures of probability.
Table 1: shows a frequency table for Serum cholesterol level in normal men 40-59 years old.

Serum cholesterol	*Frequency*	*R.F%*	*C.F%*
120-139.5	10	**1.0**	**1.0**
140-159.5	21	2.0	3.0
160-179.5	37	3.5	6.5
180-199.5	97	9.3	15.8
200-219.5	152	14.5	30.3
220-239.5	**206** أعلى تكرار	19.7	**50**
240-259.5	195	18.6	68.6
260-279.5	131	12.5	81.1
280-299.5	96	9.2	90.3
300-319.5	47	4.5	94.8
320-339.5	30	2.9	97.7
340-359.5	13	1.2	98.9
360-379.5	6	0.6	99.5
380-399.5	4	0.4	99.9
400-419.5	0	0	99.9
420-439.5	1	0.1	100.0
440-459.5	0	0	100.0
460-479.5	1	0.1	100.1
Total	1047	100.1	

Note1: The distribution is symmetric and uni model (فترة واحدةتحمل اعلى تكرار).with the peak interval 220-239.5 mg/100 ml.

Note 2: from cumulative percent column we see that 50% of these normal men had cholesterol level measuring less than 240 mg/100 ml.

Ex1: What would be the probability that his serum cholesterol measurement was in the range of 160-179?

Solution: from table 1 (37 of the 1047 have levels in this range).
The chance of selecting one of the 37 is:
(37/1047) × 100 = 3.5% as in *R.F* % column. This demonstrates the simple definition of probability.

Ex2: if you were to select to select a normal male aged 40-59 at random from the general population from these 1047 drawn, what is the probability that his cholesterol value would be less than 200?
Solution:

$$P = \frac{10+21+37+97}{1047} = 15.8\% \quad \text{(or directly from } R.F \text{ column).}$$

The probability (P): That a particular outcome (event) will occur is the ratio of No. of times the outcome can possibly occur to the total possible No. of outcome.
Note: any probability value will lie between(0 &1).because the percent R.F of the whole table add up to 100 , The probability for all class intervals would up to 1.0 ,hence any probability value will lie between 0&1.

Calculating Probability:

Addition of probability:

The probability of a single event can be calculated by enumerating the No. of ways the event can occur & dividing by total No. of events. This can be extended easily to calculating the probability of more than one event.

Ex3: What is the probability that a person drawn at random would have cholesterol level **either** below 160 mg/100 ml **or** 340 mg/100 ml or greater? Referring to Table 1.
This either/or situation can be satisfied by drawing on of the 21+10=31 persons with level below 160 mg/100 ml. or one of 13+6+4+1+1= 25 persons with level of 340mg/ 100 ml or greater.
The desired probability:

$$P = \frac{31}{1047} + \frac{25}{1047} = 0.030 + 0.024 = 0.054 = 5.4\%.$$

Note **1**: We can say there is a 5.4% chance that a person selected at random from among the 1047.will have blood cholesterol level in one of the specified ranges. The two conditions specified ***can not*** be satisfied at the same time by one person. That is; a person with cholesterol level below 160 mg/100 ml ***cannot*** at the same time, have a level greater than or equal to 340mg/ 100 ml.

Note 2: When two or more conditions ***cannot*** fulfill at the same time, they are said to be ***mutually exclusive***.
We may say that if two events, one denoted by A and the other by B, are mutually exclusive, then:

P (A or B) = P (A) + P (B).

Probability of either A occurring or B occurring is equal to the probability of A plus the probability of B.

In the foregoing example:
A is the event with condition below 160 mg/100 ml.
B is the event with condition 340mg/ 100 ml. or greater. Then:

$P(A) = \frac{31}{1047} = 0.030$, $P(B) = 25/1047 = 0.024$

$P(A \text{ or } B) = 0.030 + 0.024 = 0.054 = 5.4\ \%$

When events are **not** mutually exclusive, but can occur simultaneously. The probability of their joint occurrence must be taken into account.

Ex 4: A survey of the relative urgency of dental & medical needs by source of financial support (**table 2).**

	Most urgent need		
Source of funds	Dental	Medical	Total
public	**470**	280	750 (A)
private	110	140	250
Total	580(B)	420	1000

The data in Table 2. Will be used to illustrate the computation of probability where two events can occur jointly.

Let A be the event that the person selected is supported by public founds Verify that:

P (A) = 750/1000 = 0.75.

Let B be the event that the most urgent needs of the person selected is dental. Verify that:

P (B) = 580/1000 = 0.58.

P(A or B) is not equal to the sum of P(A) & P(B) since this would be greater than 1.

The difficulty here is that the people who are **both** supported by public funds & have urgent dental needs have been counted Twice.

470 + 280 = 750 public fund.

470 + 110 = 580 urgent dental.

The 470 people simultaneously satisfy both condition have been counted twice. They must subtract once. The correct number of people in the sample who satisfy one or other of the conditions, or both, is then given by:

750 + 580 **- 470** = 860 Therefore:

P (A or B) = 860/1000 = 0.860

It is useful to rewrite this in extended form:

P (A or B) = (750+580 **– 470**) / 1000

= (750/1000) + (580/1000) – (470/1000).

General form of probability:

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and })}$$

This is the most general form for the probability of either or both of two events.” Known as the addition rule of probability”.

Conditional probability

Conditional probability is the probability of some event *A*, given the occurrence of some other event *B*. Conditional probability is written $\mathrm{P}(A \mid B)$, and is read "the probability of *A*, given *B*". It is defined by :

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}.$$

If $\mathrm{P}(B) = 0$ then $\mathrm{P}(A \mid B)$ is formally undefined by this expression. However, it is possible to define a conditional probability for some zero-probability events using a σ-algebra of such events (such as those arising from a continuous random variable).
For example, in a bag of 2 red balls and 2 blue balls (4 balls in total), the probability of taking a red ball is $1/2$; however, when taking a second ball, the probability of it being either a red ball or a blue ball depends on the ball previously taken, such as, if a red ball was taken, the probability of picking a red ball again would be $1/3$ since only 1 red and 2 blue balls would have been remaining.

Inverse probability

In probability theory and applications, **Bayes' rule** relates the odds of event A_1 to event A_2, before (prior to) and after (posterior to) conditioning on another event B. The odds on A_1 to event A_2 is simply the ratio of the probabilities of the two events. When arbitrarily many events A are of interest, not just two, the rule can be rephrased as **posterior is proportional to prior times likelihood**, $P(A|B) \propto P(A)P(B|A)$ where the proportionality symbol means that the left hand side is proportional to (i.e., equals a constant times) the right hand side as A varies, for fixed or given B .

Summary of probabilities

Event	Probability
A	$P(A) \in [0,1]$
not A	$P(A^c) = 1 - P(A)$
A or B	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B) \qquad \text{if A and B are mutually exclusive}$
A and B	$P(A \cap B) = P(A\|B)P(B) = P(B\|A)P(A)$ $P(A \cap B) = P(A)P(B) \qquad \text{if A and B are independent}$
A given B	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B\|A)P(A)}{P(B)}$